Design of Finite Networks for Uniform Frequency Characteristic - Case 33066

MM-1680

November 3, 1926

MEMORANDUM FOR FILE

In the usual method of design for selective networks of the filter type the individual meshes are considered to be part of an infinite series of similar structures, and the network designed on this basis. The resultant structure is then terminated by impedances which are a close approximation to those assumed in determining the elements of the network. The results obtained by this method are in general satisfactory, although theoretically there must be peaks in the characteristic near the critical frequency or cutoff. In certain mechanical networks such as phonograph reproducers, these inherent peaks are of importance due to the application of constant velocity drive at the input and also to the low internal damping of most mechanical structures.

This memorandum describes a method of designing networks of the same configuration of elements and with a similar characteristic to the filter type, which does not depend upon considering the network as part of an infinite line and, therefore does not give the irregular characteristic near the critical frequency. A representative case will be considered in detail first and a discussion of other cases taken up later.

Consider the network of Figure 1:

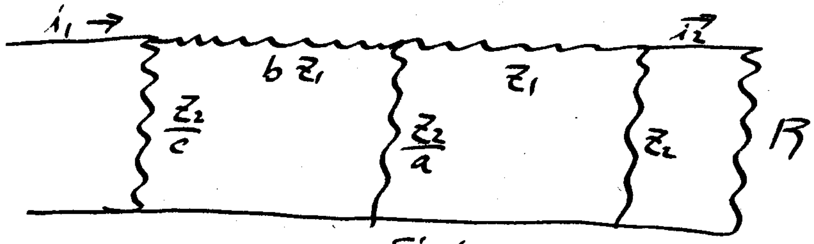

Fig. 1

Assume that the network is of the "constant K" type, that is, that $Z_1Z_2 = K^2$, then the ratio of current in to current out is

$$\frac{I_1}{I_2} = 1 - A\frac{X_1}{X_2} + B\left(\frac{X_1}{X_2}\right)^2 + j\frac{R}{\sqrt{X_1X_2}}\sqrt{\frac{X_1}{X_2}}\left[C - (A+B)\frac{X_1}{X_2} + B\left(\frac{X_1}{X_2}\right)^2\right]$$

where $Z_1 = jX_1 \qquad Z_2 = -jX_2$ and

$A = (a + bc + c) \qquad B = abc \qquad C = (1 + a + c)$

Considering the absolute magnitude of the current ratio we have

$$\left(\frac{I_1}{I_2}\right)^2 = 1 + A^2\left(\frac{X_1}{X_2}\right)^2 + B^2\left(\frac{X_1}{X_2}\right)^4 - 2A\left(\frac{X_1}{X_2}\right) + 2B\left(\frac{X_1}{X_2}\right)^2 - 2AB\left(\frac{X_1}{X_2}\right)^3$$

$$+ \frac{R^2}{X_1X_2}\left[C^2\left(\frac{X_1}{X_2}\right) + (A+B)^2\left(\frac{X_1}{X_2}\right)^3 + B^2\left(\frac{X_1}{X_2}\right)^5 - 2C(A+B)\left(\frac{X_1}{X_2}\right)^2\right.$$

$$\left. + 2BC\left(\frac{X_1}{X_2}\right)^3 - 2B(A+B)\left(\frac{X_1}{X_2}\right)^4\right] = 1 + M^2B^2\left(\frac{X_1}{X_2}\right)^5$$

where $M^2 = \frac{R^2}{X_1 X_2}$, provided that:

$$A^2 + 2B = 2M^2C(A + B)$$
$$2A = C^2M^2$$
$$2AB = M^2\left[(A + B)^2 + 2BC\right]$$
$$B = 2M^2(A + B)$$

It is possible to eliminate A, B, and C from these four equations, giving the cubic equation in M^2,

$$M^6 - \frac{3}{2}M^4 + \frac{5}{8}M^2 - \frac{5}{64} = 0$$

The three roots of this equation are readily found

$$M_1^2 = \frac{1}{4}\ ;\ M_2^2 = \frac{\sqrt{5}}{8}(\sqrt{5} + 1);\ M_3^2 = \frac{\sqrt{5}}{8}(\sqrt{5} - 1)$$

By trial it may be seen that M_1^2 and M_2^2 do not give a physical solution of the problem. Using the root M_3^2 we find that:

$$A = \frac{2\sqrt{5}}{\sqrt{5} - 2} \qquad B = \frac{10}{\sqrt{5} - 2} \qquad C = 2\left(\frac{\sqrt{5} - 1}{\sqrt{5} - 2}\right)$$

Solving for the fundamental constants a, b, and c we find

$$a = 2\sqrt{5} \qquad b = \frac{1}{\sqrt{5}(\sqrt{5} - 2)} \qquad c = 5$$

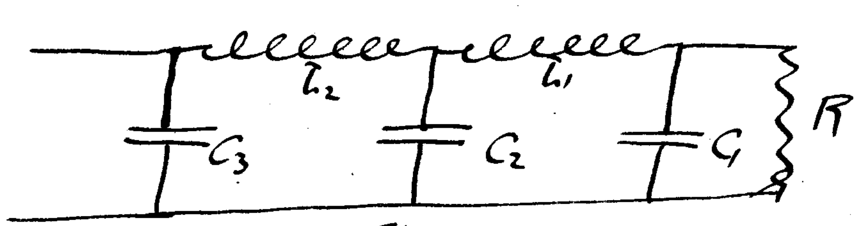

Fig 2.

Under these conditions:

$$\left(\frac{I_1}{I_2}\right)^2 = 1 + \frac{100\sqrt{5}}{(\sqrt{5}-1)^2}\left(\frac{X_1}{X_2}\right)^5$$

If we take $X_1 = wL_1$ $\qquad X_2 = \frac{1}{wC_1}$

$$\left(\frac{I_1}{I_2}\right)^2 = 1 + \frac{100\sqrt{5}}{(\sqrt{5}-1)^2} w^{10} L_1^{\,5} C_1^{\,5} = 1 + \left(\frac{w}{w_o}\right)^{10}$$

where $w_o^{\,10} = \dfrac{(\sqrt{5}-1)^2}{100\sqrt{5}\, L_1^{\,5} C_1^{\,5}}$

We have also $M^2 = R^2 \dfrac{C_1}{L_1} = \dfrac{\sqrt{5}(\sqrt{5}-1)}{8}$

Solving the last two equations for L_1 and C_1, we find

$$L_1 = \frac{2R}{\sqrt{5}\, w_o} \qquad C_1 = \frac{\sqrt{5}-1}{4w_oR}$$

Applying the constants a, b, and c to determine the other constants we have,

$$L_2 = b\, L_1 = \frac{2R}{5(\sqrt{5}-2)w_o}$$

$$C_2 = a\, C_1 = \frac{\sqrt{5}(\sqrt{5}-1)}{2\, w_o\, R}$$

$$C_3 = c\, C_1 = \frac{5(\sqrt{5}-1)}{4w_oR}$$

Referring to figure 2 and finding the numerical values of the constants, we have the following formulae. The corresponding values as determined by the filter formulae are given in parentheses.

$$L_1 = \frac{R}{1.117\,w_o} \qquad \left(\frac{R}{.50\,w_o}\right)$$

$$L_2 = \frac{R}{.591\,w_o} \qquad \left(\frac{R}{.50\,w_o}\right)$$

$$C_1 = \frac{1}{3.24 w_o R} \qquad \left(\frac{1}{w_o R}\right)$$

$$C_2 = \frac{1}{.724 w_o R} \qquad \left(\frac{1}{.50\,w_o R}\right)$$

$$C_3 = \frac{1}{.647 w_o R} \qquad \left(\frac{1}{w_o R}\right)$$

The characteristic of the above network is given by

$$TU = 10 \log_{10} \left[1 + \left(\frac{f}{f_o}\right)^{10}\right] = 10 \log_{10} \left\{\frac{I_1}{I_2}\right\}^2$$

When $f = f_o$ there is a loss of approximately 3 TU. With $f < f_o$ the loss is negligibly small, but when $f > f_o$, the loss increases rapidly, although not as rapidly as in the case of the filter type of structure. It should be noted that the values of the elements are much different from the corresponding filter values. A plot of the relative characteristics is given on an attached sketch. The curves are computed values and neglect internal dissipation in the network which would reduce the peaks in the filter characteristic to a considerable extent.

As an illustration, a phonograph reproducer will be designed by the method described. A reproducer has a series compliance due to the edge of the diaphragm which it is necessary to take into account. By the method of impedance transforming networks, it may be shown that the structure of figure 3 is identical to that of figure 2.

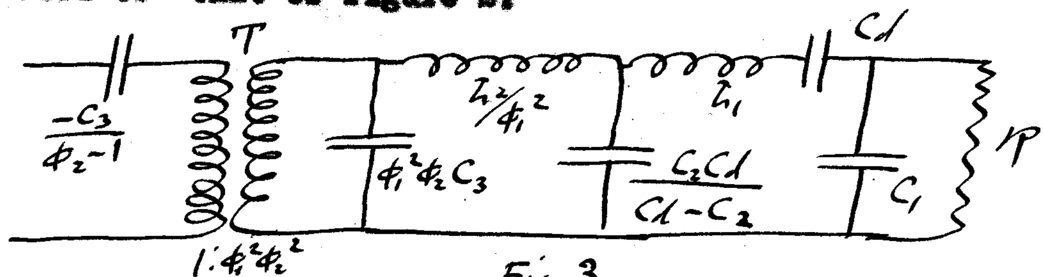

Fig. 3.

provided that $\Phi_1 = \dfrac{C_d}{C_d - C_2}$

and $\Phi_2 = \dfrac{1 - \frac{C_2}{C_d}}{1 - \frac{C_2}{C_d}\left(1 + \frac{C_3}{C_2}\right)}$

and where T is an ideal transformer of the ratio shown.

Figure 3 represents the equivalent circuit of a phonograph reproducer, except for the negative compliance in series with the input. Since the solution is simply for the ratio of input to output velocity or current, this factor does not enter into the solution and may be neglected.

We start by assuming a diaphragm of the conical type of one and three quarters inches effective diameter weighing .20 grams. This is the approximate mass of a duralumin diaphragm of .0017 inch thickness. We take further the value of C_d the compliance of the diaphragm to be $.20 \times 10^{-6}$, and select 4000 cycles as the frequency corresponding to the point where the structure is to have three TU loss.

By the relation between L_1 and R, we find R to be equal to 5620 ohms. Substituting this in the proper formulae, we have:

$$L_2 = .38 \text{ gms.}$$

$$C_1 = .0022 \times 10^{-6}$$

$$C_2 = .0098 \times 10^{-6}$$

$$C_3 = .0110 \times 10^{-6}$$

$$\phi_1 = 1.05$$

$$\phi_2 = 1.06$$

$$\phi_1^2 \phi_2 C_3 = .0129 \times 10^{-6}$$

$$L_2/\phi_1 = .345 \text{ gms}$$

$$\frac{C_2 C_d}{C_d - C_2} = .0103 \times 10^{-6}$$

The area of the diaphragm is 15.5 square centimeters. A horn to supply a load of 5620 ohms on a diaphragm of this area must have a throat opening of $\frac{41 \times 15.5^2}{5620} = 1.75$ square centimeters of a diameter of 1.5 centimeters.

Next considering the compliance of the air chamber which is required to be $.0022 \times 10^{-6}$, the required formulae for the volume of the air chamber is

$$V = 1.4 \times 15.5^2 \times .0022 = .74 \text{ cubic centimeter.}$$

The average depth of the chamber would, therefore, be $\frac{.74}{15.5}$ = .048 centimeter or .019 inches.

The effective mass of the needle arm at the diaphragm should be $\frac{L_2}{e_1^2}$ or .545 gms. which is easily realized. Since the effective compliance of the needle at the diaphragm is $.0129 \times 10^{-6}$ if l_1 represents the distance from the pivot point of the needle arm to the center of the diaphragm and l_2 the distance from the pivot point to the needle point, the ratio of $\frac{l_1}{l_2}$ is given by $(\frac{l_1}{l_2})^2 = \frac{.0129 \times 10^{-6}}{C_n}$ where C_n is the compliance of the needle point. With the average needle this ratio is approximately unity. A reproducer constructed along these lines would have theoretically a perfectly smooth characteristic, the reflection peaks which occur normally in the filter type are not present. It is of interest that the network cannot be split up into sections having the same cutoff frequency and impedance. In other words, it is not a "matched impedance" structure.

A two section network has been selected as being sufficiently general to show the method of solution. The same general method applies to simpler and more complicated structures, the solution of some others are given on the attached sketches.

The illustrative example considered above gives the solution for the ratio of input to output current, since this seems to be of more immediate practical interest. An electric network usually requires the solution for the case of a constant voltage in series with an output impedance connected to the input of the network. This condition would require the equations of the voltage divided by the current in the load to be treated as above. It is ordinarily easier, however, to make use of a simple theorem which can readily be proved, that the effect of a constant voltage E in series with an impedance Z and the network is the same as a current $I = \frac{E}{Z}$ into a parallel combination of the network and the impedance Z. If, as is usually the case, Z is a pure resistance, the solution of this case reduces to the case treated above for the ratio of two currents, with the additional complication of a resistance shunted across the input terminals of the network. If Z is not a resistance the method still applies, but here the variation of the input current $\frac{E}{Z}$ must be taken into account.

Another possible field of application depends upon an equivalent method of voltage ratios, suggested by Mr. C. H. Stevenson. It may readily be shown that if any given network terminated by the impedance Z, at the end of B, then the ratio of the current into end A to the current in the impedance Z is the same as the ratio of the two voltages $\frac{E_b}{E_a}$ where E_b is a voltage applied in series with the impedance Z and E_a is the open circuit voltage at the A end of the network. As an example, if a voltage E_b is applied in series with the resistance R of figure 2, there will be a certain voltage E_a across the condenser C_2. The ratio of the voltages $\frac{E_b}{E_a}$ will be the same for all frequencies as the ratio $\frac{I_1}{I_2}$ for the network.

One obvious field of application for this is in the case of a network where it is desired to leave the receiving end open and maintain constant voltage across the terminals, such as a vacuum tube amplifier for example, where the input circuit may be considered as a capacity. Another interesting result is that an electromagnetic recorder may theoretically be given a uniform response characteristic with no mechanical damping whatsoever. This comes about from the electromechanical network transformation, whereby series masses are replaced by shunt capacities, shunt compliances by series inductances, velocity by voltage and force by current.

In a recorder we are interested in maintaining a constant velocity in the terminal series mass, this is equivalent to maintaining a constant voltage across the terminal shunt condenser of the equivalent electric network of the electromechanical structure. Hence, the solution given above may be applied to determine the constants of a recorder with substantially uniform response characteristic and with no mechanical damping. Reference may be made to a Memorandum for File, MM-1168, "Equivalent Circuits for Magneto Mechanical Networks", Case 32184, dated January 2, 1925, for details of the equivalent electric circuit of an electromechanical network.

The same general principles may be applied to electromechanical structures to solve for the condition giving constant amplitude instead of constant velocity. This condition was considered in some detail in a Memorandum for File, MM-1812, "Constant Amplitude Motor Elements", Case 14212, dated March 19, 1925.

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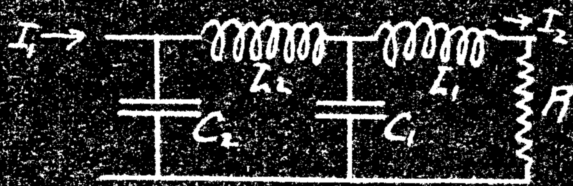
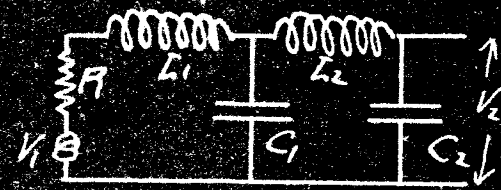
FINITE NETWORKS WITH NO RESONANT PEAKS.

1. One and one half section.



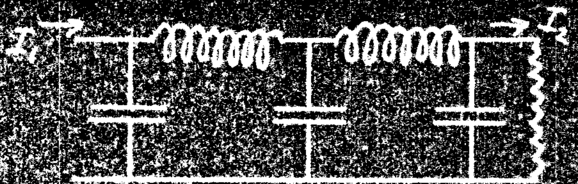


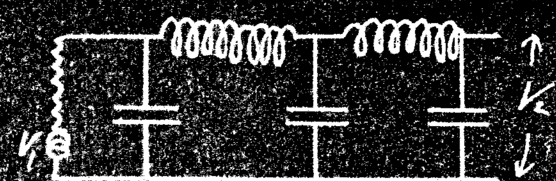
$$\left|\frac{I_1}{I_2}\right|^2 = \left|\frac{V_1}{V_2}\right|^2 = 1 + \left(\frac{f}{f_0}\right)^8$$

$$L_1 = \frac{R}{2\pi f_0 \sqrt{2\sqrt{2}(\sqrt{2}+1)}} \qquad C_1 = \frac{1}{\pi f_0 \sqrt{\sqrt{2}+2}}$$

$$L_2 = \frac{R(\sqrt{2}+1)}{4\pi f_0}\sqrt{\frac{\sqrt{2}+1}{\sqrt{2}}} \qquad C_2 = \frac{1}{\pi f_0 R}\sqrt{\frac{\sqrt{2}}{\sqrt{2}+1}}$$

2. Two section.



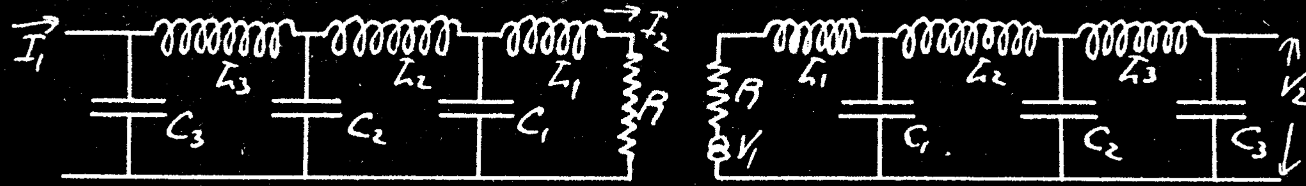


$$\left|\frac{I_1}{I_1}\right|^2 = \left|\frac{V_1}{V_2}\right|^2 = 1 + \left(\frac{f}{f_0}\right)^{10}$$

$$L_1 = \frac{R}{\sqrt{5}\,\pi f_0} \qquad C_1 = \frac{\sqrt{5}-1}{8\pi f_0 R} \qquad C_3 = \frac{5(\sqrt{5}-1)}{8\pi f_0 R}$$

$$L_2 = \frac{R}{5(\sqrt{5}-2)\pi f_0} \qquad C_2 = \frac{\sqrt{5}(\sqrt{5}-1)}{4\pi f_0 R}$$

3. Two and one half section



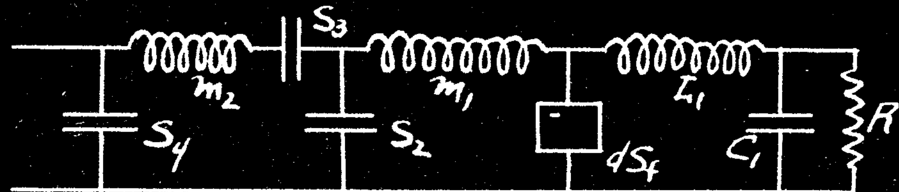
$$\left|\frac{I_1}{I_2}\right|^2 = \left|\frac{V_1}{V_2}\right|^2 = 1 + \left(\frac{f}{f_0}\right)^{12}$$

$$L_1 = \frac{(\sqrt{3}-1)R}{4\sqrt{2}\,\pi f_0} \qquad C_1 = \frac{(\sqrt{3}-1)^2}{\sqrt{2}\,\pi f_0 R}$$

$$L_2 = \frac{R}{3\sqrt{2}(\sqrt{3}-1)^3 \pi f_0} \qquad C_2 = \frac{3(\sqrt{3}-1)}{2\sqrt{2}\,\pi f_0 R}$$

$$L_3 = \frac{\sqrt{2}\,R}{3(\sqrt{3}-1)^2 \pi f_0} \qquad C_3 = \frac{3(\sqrt{3}-1)}{2\sqrt{2}\,\pi f_0 R}$$

4. Same structure transformed into magneto-mechanical network.



$$m_2 = 2(\sqrt{3}-1)\,m_1$$

$$dS_4 = \frac{3(\sqrt{3}-1)}{2}(2\pi f_0)^2 m_1$$

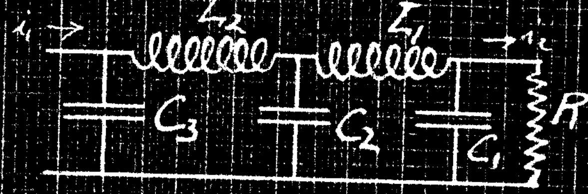
$$S_2 = (\sqrt{3}-1)^2(2\pi f_0)^2 m_1$$

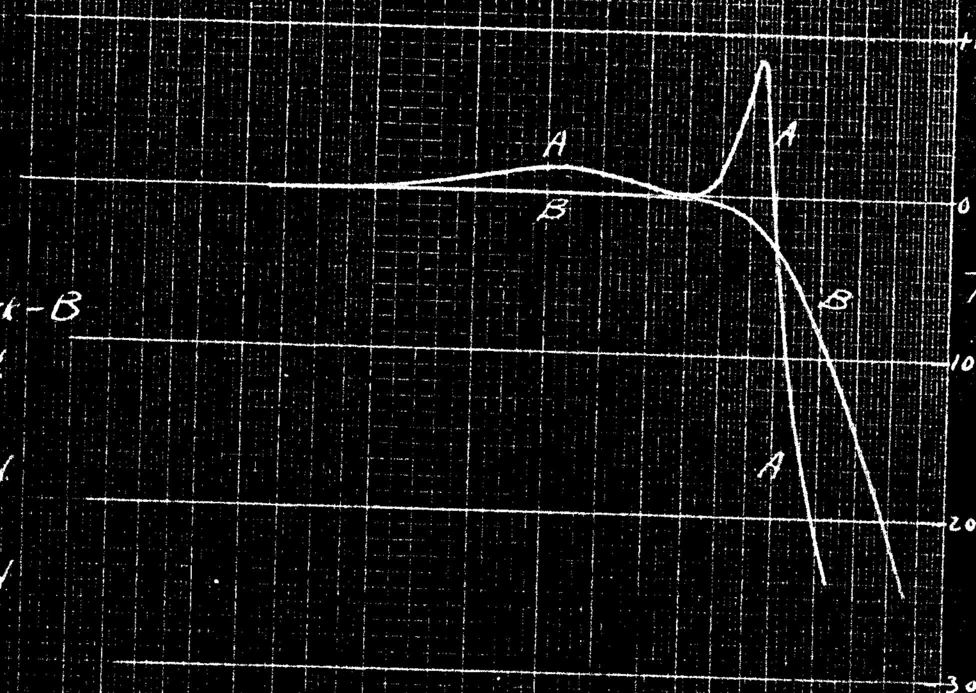
$$S_4 = (\sqrt{3}-1)^2(2\pi f_0)^2 m_1 - S_3$$

$$L_1 = \frac{(\sqrt{3}-1)^2 R}{\sqrt{2}\,\pi f_0}$$

$$C_1 = \frac{(\sqrt{3}-1)}{4\sqrt{2}\,\pi f_0 R}$$

COMPUTED CHARACTERISTICS OF TWO SECTION NETWORKS.





	Filter-A	Special Network-B
C_1	.0637 mfd	.0197 mfd.
L_1	31.8 mh.	14.2 mh.
C_2	.1274 mfd.	.0881 mfd.
L_2	31.8 mh	27.0 mh.
C_3	.0637	.0985 mfd

$$TU = 20 \log_{10} \frac{i_2}{i_1}$$

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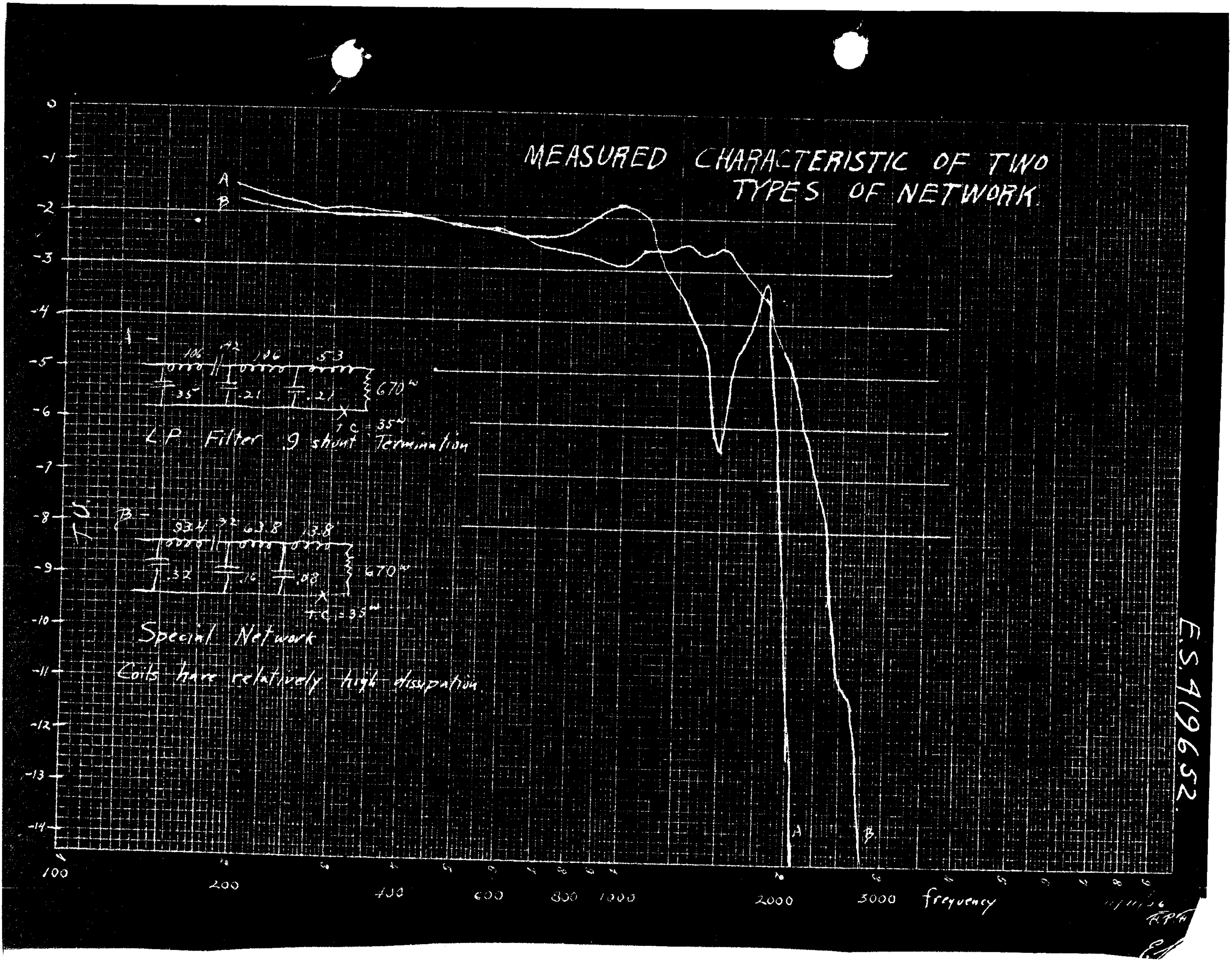

MEASURED CHARACTERISTIC OF TWO
TYPES OF NETWORK
A
B
A -
LP Filter 9 shunt Termination
B -
Special Network
Coils have relatively high dissipation
T.U.
frequency
E.S. 419652.
R.P.H.